M.Sc. (Mathematics) (NEP Pattern) Semester-II

DSE-7 - Advanced Topics in Topology

P. Pages : 2 **GUG/S/25/15402**

Time : Three Hours * 3 0 5 1 * Max. Marks : 80

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Prove that a topological space X is completely normal iff every subspace of X is normal. **8**

b) Prove that every metric space is completely normal. **8**

OR

c) Prove that a normal space is completely regular iff it is regular. **8**

d) Prove that every countably compact metric space is totally bounded. **8**

UNIT – II

2. a) Prove that $\prod_{\lambda} X_\lambda$ is Hausdorff iff each space X_λ is Hausdorff. **8**

b) Prove that $X \times Y$ is dense-in-itself iff at least one of the spaces X and Y is dense-in-itself. **8**

OR

c) Prove that $\prod_{\lambda} X_\lambda$ is connected iff each space X_λ is connected. **8**

d) Prove that $\prod_{\lambda} X_\lambda$ is first axiom iff each space X_λ is first axiom and all but a countable number are indiscrete. **8**

UNIT – III

3. a) Prove that a subset G of Y is open in the quotient topology relative to $f : X \to Y$ iff $f^{-1}(G)$ is an open subset of X. **8**

b) Prove that every paracompact regular space is normal. **8**

OR

c) If f is a continuous, open mapping of the topological space X onto the topological space Y, then prove that the topology for Y must be the quotient topology. **8**

d) If X is a regular paracompact space and Y is regular σ - compact space, then prove that $X \times Y$ is paracompact. **8**

UNIT – IV

4. a) Prove that a topological space is Hausdorff iff limits of all nets in it are unique. **8**

b) Let $S : D \to X$ is a net in a topological space and let $x \in X$. Then prove that x is a cluster point of S iff there exists a subnet of S which converges to x in X. **8**

OR

c) Prove that a topological space is Hausdorffi ff no filter can converge to more than one point in it. **8**

d) Prove that a topological space is compact iff every ultrafilter in it is convergent. **8**

5. a) Define – **4**
i) Completely Normal Space
ii) Completely Regular Space

b) Define Tichonov topology for the product $\prod_{\lambda} X_{\lambda}$ of topological spaces. **4**

c) Define – **4**
i) Quotient Topology
ii) Paracompact space

d) Suppose $S : D \to X$ be a net and F is cofinal subset of S. If $S / F : F \to X$ converges to a Point x in X, then prove that x is a cluster point of S. **4**

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